General Biostatistics

Part 5

Common Statistical Tests

Comparing Means
of Two Groups

Outline

- Hypothesis testing
- Confidence interval estimation
  - Analysis of two independent groups
  - Analysis of pair-matched data

Two Groups

- Calculate test statistic for hypothesis test (Ho: $\mu_1 - \mu_2 = 0$)
- Calculate 95% confidence interval for the true difference in population means

Two Groups

- Hypothesis test comparing means of two groups
  - z test
    - Known population variances in both groups
  - t test
    - Unknown population variances (equal or unequal)

Two Groups

- In general

$$\text{test statistic} = \frac{(\text{sample statistic - hypothesized value})}{\text{standard error of the sample statistic}}$$

- Need to know
  - calculated sample statistic
  - hypothesized value of population parameter
  - standard error

Example: Clinical Trial

Patients on Drug	Patients on Placebo
19	22
11	18
14	17
17	19
23	25

Example: Clinical Trial

- 5 patients in drug group, 5 patients in placebo group of trial to decrease anxiety
- Calculate $\overline{X}_1 = 16.8$ $\overline{X}_2 = 20.2$
- $s_1 = 4.6$, $s_2 = 3.3$
- Difference in mean anxiety score between drug and placebo was 3.4 points (lower mean anxiety score with drug)

Example: Clinical Trial

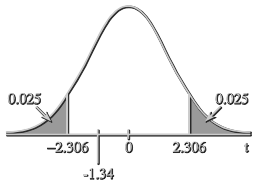
- Test statistic

$$t = \frac{(\overline{X}_1 - \overline{X}_2 - 0)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(-3.4 - 0)}{\sqrt{\frac{(4.6)^2}{5} + \frac{(3.3)^2}{5}}} = -1.34$$

- Statistical decision?
- p-value?
- 95% confidence interval?

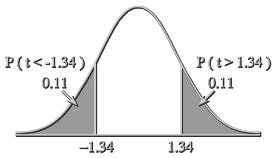
Example: Clinical Trial

- Test statistic
- t = - 1.34,
- df = $(n_1-1) + (n_2-1)$
  = (5-1) + (5-1) = 8
- Statistical decision?
  - Compare to tabled t-statistic with 8 df, α = 0.05/2 of 2.306
  - -1.34 > -2.306; do not reject Ho



Example: Clinical Trial

- p-value?
- One-sided p =0.11
- Two-sided p=0.22



Example: Clinical Trial

- 95% confidence interval?
- (-9.2, 2.4)

$$(\overline{\overline{X}}_1 - \overline{\overline{X}}_2) \pm t_{0.05/2,df=8}\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$= -3.4 \pm 2.306\sqrt{\frac{(4.6)^2}{5} + \frac{(3.3)^2}{5}} = (-9.2, 2.4)$$

Pair - Matched Data

- Two samples are not independent
- Related or paired observations (e.g., pre-post design)
- Can consider as a simple random sample from a normally distributed population of differences between paired observations

Pair - Matched Data

- Calculate test statistic for hypothesis test (Ho: $\mu_d = 0$)
- Calculate 95% confidence interval for the true mean difference

Pair - Matched Data

- In general

$$\text{test statistic} = \frac{(\text{sample statistic - hypothesized value})}{\text{standard error of the sample statistic}}$$

- Need to know
  - calculated sample statistic
  - hypothesized value of population parameter
  - standard error

Example: Cross-over Drug Trial

Patient	Week 1 Drug	Week 2 Placebo	Week 1-Week 2
1	19	22	-3
2	11	18	-7
3	14	17	-3
4	17	19	-2
5	23	25	-2

Example: Cross-over Drug Trial

- 5 patients in a cross-over placebo/drug trial to decrease anxiety
- Calculate $\bar{d}$ =-3.4 s_d = 2.1
- Mean difference in anxiety score between drug and placebo was 3.4 points (decrease in anxiety score with drug)

Example: Cross-over Drug Trial

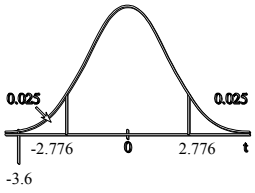
- Test statistic

$$t = \frac{(\bar{d} - \mu_d)}{\frac{s_d}{\sqrt{n}}} = \frac{(-3.4 - 0)}{\frac{2.1}{\sqrt{5}}} = -3.6$$

- Statistical decision?
- p-value?
- 95% confidence interval?

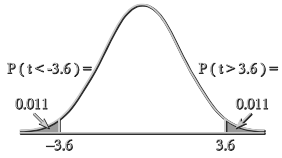
Example: Cross-over Drug Trial

- Test statistic
- t = - 3.6,
- df = (n-1) = 5-1 = 4
- Statistical decision?
  - Compare to tabled t-statistic with 4 df, α = 0.05/2 of 2.776
  - -3.6 < -2.776; reject Ho



Example: Clinical Trial

- p-value?
- One-sided p =0.011
- Two-sided p=0.022



Example: Clinical Trial

- 95% confidence interval?
- (-6.0, -0.8)

$$\bar{d} \pm t_{0.05/2,df=4}\sqrt{\frac{s_d^2}{n}}$$

$$= -3.4 \pm 2.776\sqrt{\frac{2.1^2}{5}} = (-6.0, -0.8)$$

Summary

- Use sampling distribution for hypothesis test and confidence interval construction
  - Two groups - continuous data
  - Pair-matched continuous data